A MARKOVIAN ANALYSIS OF MIGRATION DIFFERENTIALS[1]

Andrei Rogers, University of California

1. Introduction

Over the past twenty years, quantitative models of internal migration have received considerable attention in the social sciences, particularly in the areas of sociology and demography. A vast amount of data have been collected, and numerous mathematical models have been proposed to account for apparent empirical regularities. These indicate that migration is a clearly patterned nonrandom phenomenon which is subject to scientific explanation and, therefore, perhaps ultimately may be forecast with a reasonable degree of accuracy.

Internal migration may be approached from two different points of view: from the point of view of migration streams and from the point of view of migration differentials. These are not mutually exclusive conceptualizations, but each concentrates on a particular aspect of migration. Migration stream analysis focuses on the volume and direction of place-to-place movements. The analysis of migration differentials selects as its principal subject of inquiry the differences in the characteristics of migrants and nonmigrants and the differences between migrant sub-groups. Whereas the analysis of streams is concerned primarily with the effect that variations in environmental conditions at origins and destinations have on volumes of flow, the study of differentials is concerned with the traits of migrants in various age-sex-income-race classifications. Thus the problem shifts from that of accounting for changes in flow patterns to explaining in what respects migrants differ from the general population. In short, differential migration is concerned with the study of those migrant categories which have a disproportionately greater or smaller percentage of migrants than is found in the population as a whole.

The definitive work on migration differentials continues to be that of Dorothy S. Thomas, whose exhaustive findings on this topic were published almost thirty years ago.[2] Since that time several significant analyses of migration differentials have appeared. Bogue and Hagood, by cross-classifying stream characteristics, simultaneously consider the joint effects of income, age, occupation, employment, marital status and education on migration.[3] Beshers and Nishiura suggest a theory of internal migration differentials.[4] The principal hypotheses which consistently reappear in these and other studies are:

1. Young adults are the most mobile segment of the population.
2. Males tend to be more migratory than females.
3. Unemployed persons are more likely to move than employed persons.
4. Whites move more than non-whites.
5. Professionals are among the most mobile elements of the population.

Paralleling the growing interest in quantitative analysis of migration phenomena has been the emergence of Markov chain theory as a methodological tool for analyzing social, industrial and geographic mobility. Markov chains have been used to examine intergenerational mobility,[5] to study the movement of workers between industries,[6] and to project future population totals for Census Divisions in the United States.[7] By and large, however, the empirical results have been disappointing. What at first appeared as a powerful new technique for temporal analysis has been found to be generally inapplicable in much of sociological and demographic research. Fundamentally, the discouraging results stem from the restrictive assumption of unchanging movement probabilities. Such an assumption, of course, is unrealistic in light of our knowledge concerning mobility in general and interregional migration in particular. Transition probabilities vary over time as well as over space. Moreover they are dependent on differential socioeconomic, demographic and political situations at origins and destinations. Thus one may justifiably conclude that Markov chain analysis may be more useful in analyses of past migration flows and of very little practical use in efforts to forecast future place-to-place movements. However, though of limited utility in temporal analysis, it appears that Markovian concepts do provide useful indices for purposes of differential analysis. Thus despite its limited success in accounting for interregional migration streams, Markov chain theory does supply useful insights concerning the observed differential behavior of a population of migrant cohorts at a given point in time.

This paper describes an investigation of migration differentials in California. The data are the U.S. Census reported flows for the 1955-1960 time period and supplementary estimates provided by a recent study completed for the California State Development Plan.[8] The method of analysis utilizes the Markovian concepts of transition matrices, mean first passage times and equilibrium distributions.

2. Markovian Analysis of Migration Differentials

Consider an interregional system of m regions and a population composed of n cohorts. Define a cohort as a group of persons who behave independently but according to an identical migration structure. That is, assume that a member of cohort r behaves independently of all other members and according to an m by m transition matrix P_r. Then we may estimate each element of P_r by means of observed proportions taken over a cohort class, i.e.,

$$ {}_r p_{ij} = \frac{{}_r k_{ij}}{\sum_{j=1}^{m} {}_r k_{ij}}, \quad (r = 1, 2, \ldots, n) \quad (i, j = 1, 2, \ldots, m) $$

where k_{ij} = the number of people, who during a specified time period, moved from region i to region j.

With cohort-specific data on migration propensities, we may begin to study the changes of state that a single individual is likely to undergo in light of the transition structure of his cohort class. More specifically, for each cohort, we may identify current movement characteristics and thereby establish a series of intra-cohort contrasts. Three properties of transition structures serve as particularly useful indices: the cohort's transition matrix, the associated mean first passage time matrix and the equilibrium vector.

Transition Matrices

Cohort-specific transition matrices provide a great deal of information about the mobility of migrant classes. In particular, their diagonal elements provide an immediate dimension along which we may contrast the degree of overall mobility of different migrant groups. For example, consider a hypothetical system of only two regions, A and B, and a population divided into two broad cohort classes, white and non-white. Let us suppose that if an individual, in the white cohort class, is in region A there is a 50 per cent chance that he will move to region B during the unit time interval. If the person is currently in region B, however, with probability 1/4 he will move to A during the same time period. Assume, further, that for the non-white cohort class the corresponding probabilities are 1/4 and 1/5. In matrix form we have then:

$$ P_W = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{pmatrix} \end{array} \qquad P_{NW} = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 3/4 & 1/4 \\ 1/5 & 4/5 \end{pmatrix} \end{array}. $$

Immediately we observe that the diagonal elements of P_{NW} are greater than the corresponding entries in P_W. From this we may infer that non-whites are less mobile than whites. If our interregional system contained more than two regions, we would, in addition, be in a position to compare the relative "attraction" of alternate destinations for different migrant cohorts.

Mean First Passage Times

Frequently it is desirable to study the length of time that it takes an average individual to move from state i to state j for the first time. The distribution describing this random variable is called the first passage time distribution. Its mean is commonly referred to as the mean first passage time.

Turning to our two-region example, consider the probability that an individual currently in region A will move to region B, for the first time, in n time periods. Denote this probability by $g_{AB}^{(n)}$ and begin with n equal to 1. Then,

$$ g_{AB}^{(1)} = p_{AB} $$

$$ g_{AB}^{(2)} = p_{AA} \cdot p_{AB} $$

and by substitution

$$ g_{AB}^{(2)} = p_{AA} g_{AB}^{(1)} $$

The above equations merely state than an individual's probability of going from A to B, for the first time, in one time period is p_{AB} (by definition), and the probability of doing this in two steps is the product of the probability of remaining in A during the first time period and the probability of moving to B during the second time period.

Extending the argument to the general case, for this two-region example, we have:

$$ \begin{aligned} g_{AB}^{(n)} &= p_{AA} \cdot g_{AB}^{(n-1)} \\ &= p_{AA} \cdot p_{AA} \cdot g_{AB}^{(n-2)} \\ &= \ldots \\ &= (p_{AA})^{n-1} \cdot p_{AB} \end{aligned} $$

This function is called the first passage time distribution. Since $p_{AA} = 1 - p_{AB}$, we have

$$ g_{AB}^{(n)} = p_{AB}(1 - p_{AB})^{n-1} $$

which is the geometric distribution with a mean

$$ m_{AB} = \frac{1}{p_{AB}} $$

This statistic is defined as the mean first passage time and represents the average number of time periods required for a person in region A to visit region B for the first time. The matrix M, consisting of entries m_{ij}, is defined as the mean first passage time matrix.

Returning to our numerical example, we find for the white cohort:

$$ g_{AB}^{(n)} = (1/2)\ (1/2)^{n-1} $$

and

$$m_{AB} = \frac{1}{1/2} = 2 .$$

In general, the mean first passage times of a Markov chain may be found by recursively applying the following equation:[9]

$$m_{ij} = p_{ij} + \sum_{k \neq i} p_{ik} m_{ki} .$$

Kemeny and Snell, however, offer a more convenient matrix formulation:

$$M = (I - Z + EZ_{dg})D$$

where

- D = a diagonal matrix with elements $d_{ii} = 1/a_i$;
- E = a matrix with all elements equal to 1;
- I = the identity matrix;
- Z = the fundamental matrix;
- Z_{dg} = the Z matrix with all off-diagonal entries set equal to 0.

The fundamental matrix, Z, is defined by the following equation:

$$Z = (I - (P-A))^{-1} ,$$

where

- P = the matrix of transition probabilities;
- A = a matrix with each row identically equal to the equilibrium vector $\underline{a}$.

The computation of the matrix of mean first passage times may be illustrated by returning to our example:

$$Z_W = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \left[\begin{pmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{pmatrix} - \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix} \right] \right\}^{-1}$$

$$= \begin{pmatrix} 5/6 & 1/6 \\ 1/12 & 11/12 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 11/9 & -2/9 \\ -1/9 & 10/9 \end{pmatrix} ,$$

and

$$M_W = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 11/9 & -2/9 \\ -1/9 & 10/9 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 11/9 & 0 \\ 0 & 10/9 \end{pmatrix} \right] \begin{pmatrix} 3 & 0 \\ 0 & 1\,1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 4 & 1\,1/2 \end{pmatrix} .$$

As a check, notice that m_{12} is again equal to 2.

Repeating the above computation for the non-white cohort, we have:

$$M_{NW} = \begin{pmatrix} 2\,1/4 & 4 \\ 5 & 1\,4/9 \end{pmatrix} .$$

Mean first passage times provide a measure of a particular kind of contiguity--one based on interchange probabilities rather than distance. Thus they may be viewed as indices of aspatial interregional distance. Let us define this aspatial measure of proximity as "migrant distance."

With reference to our two-region, two-cohort example, we may make both an intra-cohort observation and an inter-cohort contrast:

(1) "Migrant distance" from region A to region B, for both cohorts, is "shorter" than the distance from B to A. This asymmetry suggests that, on the basis of actual migrant exchange, B is "closer" to the population at A than A is to the population at B.

(2) White "migrant distance" from region A to region B is "shorter" than non-white "migrant distance" between the same two regions.

Equilibrium or Limiting State Probabilities

The transition matrix P provides a great deal of information about the Markov process described above. For example, it allows us to derive the probability that an individual currently residing in region A will be in region B after 2 years. This "event" can occur only in one of two mutually exclusive and collectively exhaustive ways:

(1) the individual remains in A during the first year and migrates to B during the

second year;

(2) the individual migrates to B during the first year and remains in B during the second year.

Therefore, for the white cohort,

$$p_{AB}^{(2)} = p_{AA}p_{AB} + p_{AB}p_{BB}$$
$$= (1/2)(1/2) + (1/2)(3/4)$$
$$= 5/8 .$$

With analogous arguments we find:

$$p_{AA}^{(2)} = (1/2)(1/2) + (1/2)(1/4) = 3/8,$$

$$p_{BA}^{(2)} = (1/4)(1/2) + (3/4)(1/4) = 5/16,$$

$$p_{BB}^{(2)} = (1/4)(1/2) + (3/4)(3/4) = 11/16.$$

These numbers can be presented in a matrix:

$$P_W^{(2)} = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 3/8 & 5/8 \\ 5/16 & 11/16 \end{pmatrix} \end{array} .$$

The matrix $P^{(2)}$ describes movement between two periods of time. Similarly $P^{(n)}$ describes movement during n time periods. It should now become apparent that the transition matrix P, in a Markov model, completely determines the character of the migration process. Therefore, it is possible to use this short term data to compare the movement patterns of different classes of individuals, to project these into the future, and to assess what are the intrinsic distributional consequences of a particular movement structure.

The essential feature of representing Markov processes by transition matrices stems from the ease with which nth order transition probabilities may be derived by matrix multiplication. In particular, the multiplication of the transition probability matrix P by itself, n number of times, yields the nth-order transition probabilities. For example, it can be shown that:

$$P^{(2)} = P \cdot P = P^2$$

and, in general,

$$P^{(n)} = P^n \quad .$$

This can be demonstrated by our example:

$$P_W = \begin{pmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{pmatrix} = \begin{pmatrix} .50 & .50 \\ .25 & .75 \end{pmatrix}$$

$$P_W^2 = \begin{pmatrix} .38 & .62 \\ .31 & .69 \end{pmatrix}$$

$$P_W^3 = \begin{pmatrix} .34 & .66 \\ .33 & .67 \end{pmatrix}$$

$$P_W^5 = \begin{pmatrix} .33 & .67 \\ .33 & .67 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix} .$$

Similarly, for the non-white cohort:

$$P_{NW}^8 = \begin{pmatrix} 4/9 & 5/9 \\ 4/9 & 5/9 \end{pmatrix} .$$

An interesting and very important feature of a class of Markov processes, defined as "ergodic" chains, is illustrated by the above matrices. It will be noted that initially the transition probabilities are different for each of the two states. That is, a migrant's destination is heavily influenced by his place of origin. However, after n powers of the transition matrix are calculated, it becomes apparent that the effect of the starting state diminishes. For example, for the white cohort this occurs when n is equal to 5. For this and larger values of n, the rows of the transition matrix are identical. This means that as n increases, $p_{ij}^{(n)}$, the probability of migrating from i to j in n years, approaches a limit p_j which is independent of i. At this point the system is said to be in "equilibrium" or to have reached a "steady state."

Comparing the equilibrium vectors of the white and non-white cohorts in our example suggests something of the long-term implications of current behavior. It is an abstract index, to be sure, since "death" is not included as a possible end-state. Nevertheless, the steady state vector may be viewed as a kind of "speedometer" which describes the ultimate consequences of the current movement pattern if it remains unchanged. Instead of assuming that the driver doesn't die and that his car continues at exactly the same speed, we assume that the migrant doesn't die and that the transition probabilities remain constant.

In our example, we note that on the basis of current trends it appears that the white cohort is favoring region B as a destination. A similar observation may be made with respect to the non-white cohort.

3. Migration Differentials in California: Some Empirical Results

According to the Census of 1960, over 2.1 million persons migrated to California between 1955 and 1960 while slightly under a million departed, thus producing a net increase of some 1.2 million people over the five-year period.[10] Origins and destinations for these migrants, by 19 State Economic Areas, have been published[11] and total age- and color-specific intrastate flows and transition matrices have been estimated.[12] For ease of exposition, however, we shall structure the discussion around selected matrices of a smaller order. In particular, we shall focus on the reduced versions which are exhibited in Tables 1 through 6.[13]

Transition Matrices

Several interesting findings are suggested by the transition matrices. These are by no means surprising and, indeed, merely support relatively well-established demographic hypotheses.

First, it is clear that the transition probabilities have not remained constant over time. The population has become much more mobile both at the interstate and the intrastate levels. Second, there are significant differences between the characteristics of white and non-white flow patterns. Non-white probabilities are considerably higher than white probabilities in urban to urban transfers and much lower in urban to suburban-rural movements. Finally, considerable differences appear to exist between the migration structures of various age groups. The most mobile age groups are the 15 to 19 and 20 to 24 age groups; the least mobile are the post-65-year age groups.

Temporal Differentials: The transition matrix for California SEA's has changed considerably over time. This is immediately apparent from even the most cursory examination of Table 1. In every instance the diagonal element of the 1935-1940 matrix is larger than the corresponding diagonal element of the 1955-1960 matrix. This points to the greater mobility of today's population. For example, for the 1935-1940 cohort, the probability that a member of the San Francisco-Oakland population moves out of that SMSA is less than .09. The corresponding figure for the 1955-1960 time interval is almost .15. The change for other SEA's is less striking, but is significant nevertheless.

Color Differentials: Two major points should be noted concerning the white and non-white transition matrices presented in Tables 2 and 6. First, the data clearly show that, on the whole, whites are more mobile than non-whites. Every diagonal element of the non-white matrix in Table 2 is larger than the corresponding element of the white matrix. For example, the probability that an individual of the non-white cohort in the Los Angeles-Long Beach SMSA moves out of that subregion during the 1955-1960 period is less than 0.6. The corresponding figure for whites is exactly twice that number.

The second finding concerns rural to urban transfers. Non-white movements are primarily urban to urban migrations. Non-white probabilities are relatively higher than white probabilities in SMSA to SMSA movements, but are much lower in SMSA to non-SMSA transfers.

Age Differentials: Tables 3, 4, and 5 highlight the age-specific mobility pattern which emerges out of an analysis of the transition matrices of the 17 age cohorts in California. Although considerable differences exist between individual SEA's, the overall distribution is unmistakable. The probability of leaving an SEA is highest for the 15- to 19- and 20- to 24-year age groups and lowest for the post-65-year age groups. The distribution is unimodal and resembles the Gamma distribution. The high values are distributed around .40 with the low values approaching zero. The maximum is attained by the South Central Coast SEA. Here the probability that an individual in the 15- to 19-year age group moves out of this SEA is almost .44.

Mean First Passage Times

Tables 7, 8, and 9 present mean first passage time matrices for six of the eight transition matrices appearing in Tables 1, 2, 4, and 5. The actual values of these "migrant distances" are quite meaningless; however, when considered in relative terms, they suggest several interesting findings concerning spatial and aspatial contiguities among California's major SMSA's.

A quick glance at the 1935-1940 and 1955-1960 mean first passage time matrices reveals changes both in intra- and inter-matrix levels. On the whole, it is clear that migrant distances declined over the twenty-year period--a reflection of increased geographical mobility. Other changes, however, are equally noteworthy. Perhaps the most noticeable is the shortening of migrant distances in relation to the distance between the Los Angeles-Long Beach and San Francisco-Oakland SMSA's. For example, whereas during the 1935-1940 period the migrant distance from Los Angeles to San Jose was over four times that of the migrant distance between Los Angeles and San Francisco, in 1955-1960 this ratio declined to two to one.

Differences both within and between the white and non-white mean first passage time matrices are quite apparent. Particularly striking are the non-white migrant distances to the San Jose SMSA. The non-white migrant distance between the San Francisco Oakland and the San Jose SMSA's, for example, is nine times the reverse distance and thirteen times the distance between the San Francisco and Los Angeles SMSA's. This probably is a reflection of the racial discrimination in San Jose's housing market.

The mean first passage time matrices for the 20- to 24- and 65- to 69-year age groups differ considerably in absolute values but are very similar in relative terms. This is an indication that, although the former age group is much more mobile than the latter age group, their movement patterns are quite similar. For example, in both matrices the distance from San Jose to Sacramento is three times that of the reverse distance.

Finally, it is interesting to note the total absence of any significant correlation between interregional highway-mileage distances (Table 10) and interregional migrant distances as measured by mean first passage times. Table 11 presents the correlations between each of the mean first passage time matrices in Tables 7, 8, and 9 and the interregional distances shown in Table 10. Clearly the spatial and aspatial measures of interregional distances are totally unrelated.

Equilibrium Distributions

The migration differentials revealed by the transition matrices in Tables 1, 2, 3, 4, 5, and 6 are readily recognizable. Differences in the propensity to move are immediately apparent. Not so obvious perhaps, are the implied distributional consequences of the various transition structures. For example, a comparison of the equilibrium vectors of the 1935-1940 and the 1955-1960 transition matrices suggests that California's share of the national population is going to taper off at a lower level than indicated by pre-World War II trends. This is not immediately apparent from a consideration of the transition matrices alone.

At more disaggregated levels, the equilibrium solutions present a detailed, quantitative picture of the spatial implications of current mobility trends. Moreover, they provide indications of temporal changes and of differentials between migrant sub-classes.

Temporal Differentials: The temporal changes in the values of the equilibrium vectors for California's population have little meaning other than as an index of the direction of changes in regional preferences over time. Perhaps the most significant finding in Table 1 is the decline in the equilibrium probabilities of the San Francisco-Oakland and Los Angeles-Long Beach SMSA's. This, however, is not an unexpected trend, especially when viewed against the increasing equilibrium probabilities of the San Jose, Sacramento and San Diego SMSA's.

Color Differentials: The most striking finding arising out of the equilibrium vectors in Table 2 is the overwhelming expected concentration of non-whites in the Los Angeles and San Francisco regions. Of the projected non-white share for California, well over half are expected to settle in the Los Angeles-Long Beach SMSA and about a fifth should locate in the San Francisco-Oakland SMSA. This is in marked contrast to the white equilibrium vector. The latter exhibits a relatively more uniform distribution, though it too shows a significant concentration in the Los Angeles subregion.

Age Differentials: Despite considerable differences between age-specific transition matrices, the equilibrium vectors of the six age groups analyzed in Tables 3, 4, and 5 are, on the whole, quite similar. The major difference appears in the California-Rest of the U.S. probability allocation. Thus, for example, whereas for the 20- to 24-year age group this division is .193-.807, for the 65- to 69-year age group the corresponding split is .138-.862. Among the five SMSA's, however, the vector does not vary substantially between age groups.

4. Conclusion

This paper has borrowed concepts from Markov chain theory to identify and analyze migration differentials. Transition matrices were used to establish the movement propensities of each migrant cohort. Mean first passage times defined aspatial measures of interregional "migrant distance." Finally, equilibrium distributions pointed to the distributional tendencies of different classes of migrants.

The basic Markovian model is conceptually simple and rests on very strict assumptions concerning human behavior. Because of this, it is an analytic system which shows only limited promise as a tool for long-term forecasting of interregional flows. However, as a technique for analyzing differential behavior during an observed period, it appears to provide insights which are not readily obtainable by other means.

5. Footnotes

1 This study began in 1964 as one of several Phase II California State Development Plan Studies conducted by the Center for Planning and Development Research at the University of California. The research was prepared under contract to the State Office of Planning and was financed in part through an urban planning grant from the Housing and Home Finance Agency, under the provisions of Section 701 of the Housing Act of 1954, as amended. The author is indebted for this financial assistance.

2 Dorothy S. Thomas, Research Memorandum on Migration Differentials (New York: Social Science Research Council, 1938), Bulletin 43.

3 Donald J. Bogue and Margaret Jarmon Hagood, Differential Migration in the Corn and Cotton Belts (Oxford, Ohio: Scripps Foundation, 1953).

4 James M. Beshers and Eleanor N. Nishuira, "A Theory of Internal Migration Differentials," Social Forces (1961), 39, pp. 214-218.

5 S. J. Prais, "Measuring Social Mobility," Journal of the Royal Statistical Society, Series A (1955), pp. 56-66; John G. Kemeny and J. Laurie Snell, Finite Markov Chains (Princeton,

N.J.: D. Van Nostrand Co., 1960), pp. 191-200.

6 Isadore Blumen, Marvin Kogan and Philip J. McCarthy, The Industrial Mobility of Labor as a Probability Process, Vol. VI, Cornell Studies of Industrial and Labor Relations (Ithaca, New York: The New York State School of Industrial and Labor Relations, Cornell University, 1955).

7 James D. Tarver and William R. Gurley, "A Stochastic Analysis of Geographic Mobility and Population Projections of the Census Divisions in the United States," Demography, II (1965), pp. 134-139. Also see: Robert McGinnis and John E. Pilger, "On a Model for Temporal Analysis," paper presented at the 58th Annual Meeting of the American Sociological Association, Los Angeles, August 29, 1963.

8 Andrei Rogers, An Analysis of Interregional Migration in California, Center for Planning and Development Research, University of California, Berkeley, California, December 1965.

9 For a derivation of this equation, see: John G. Kemeny and J. Laurie Snell, op. cit., pp. 78-80. Their notation has been retained in order to reduce possible confusion.

10 Financial and Population Research Section, California Migration: 1955-1960, California Department of Finance, Sacramento, 1964, p. 1.

11 U.S. Census of Population, 1960, Mobility for States and State Economic Areas, U.S. Bureau of the Census, Department of Commerce, 1963.

12 These were developed in the study: Andrei Rogers, Projected Population Growth in California Regions: 1960-1980, Center for Planning and Development Research, University of California, December 1965. For estimating procedures see pp. 15-17 of that study.

13 Transition matrices for the 1935-1940 time period were derived from interregional flow data reported in: Donald J. Bogue, Henry S. Shryock, Jr., and Siegfried A. Hoermann, Subregional Migration in the United States, 1935-40, Volume I: Streams of Migration (Oxford, Ohio: Scripps Foundation. Miami University, 1957).

TABLE 1. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY TIME PERIOD.*

A. 1935-1940 Total Flows

	A	B	C	F	G	CAL.	U.S.
A	.9139	.0067	.0049	.0615	.0022	.0293	.0265
B	.0575	.8529	.0056	.0121	.0034	.0459	.0226
C	.0379	.0030	.8434	.0125	.0019	.0741	.0272
F	.0096	.0012	.0013	.9215	.0058	.0242	.0364
G	.0147	.0014	.0043	.0498	.8371	.0208	.0719
CAL.	.0280	.0059	.0086	.0031	.0044	.8912	.0288
U.S.	.0009	.0001	.0001	.0033	.0004	.0020	.9932

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0387	.0050	.0053	.0688	.0065	.0479	.8279)

B. 1955-1960 Total Flows

	A	B	C	F	G	CAL.	U.S.
A	.8543	.0203	.0070	.0172	.0053	.0363	.0596
B	.0460	.8271	.0053	.0155	.0043	.0465	.0553
C	.0247	.0061	.8165	.0142	.0034	.0667	.0684
F	.0076	.0043	.0030	.8907	.0078	.0324	.0542
G	.0120	.0046	.0019	.0371	.7923	.0255	.1266
CAL.	.0209	.0099	.0109	.0327	.0078	.8538	.0640
U.S.	.0017	.0006	.0004	.0056	.0016	.0028	.9873

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0253	.0107	.0070	.0667	.0116	.0456	.8331)

*A = S.F. - Oakland; B = San Jose; C = Sacramento; F = Los Angeles; G = San Diego; Cal. = Rest of California; U.S. = Rest of the U.S.

TABLE 2. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY COLOR

A. 1955-1960 White Flows

	A	B	C	F	G	CAL.	U.S.
A	.8465	.0221	.0073	.0172	.0056	.0388	.0625
B	.0453	.8269	.0053	.0154	.0044	.0465	.0562
C	.0247	.0063	.8118	.0141	.0035	.0689	.0707
F	.0077	.0046	.0032	.8863	.0082	.0339	.0561
G	.0117	.0048	.0019	.0367	.7897	.0260	.1292
CAL.	.0208	.0102	.0110	.0326	.0080	.8526	.0648
U.S.	.0018	.0006	.0004	.0058	.0017	.0030	.9867

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0249	.0112	.0071	.0661	.0121	.0477	.8309)

B. 1955-1960 Non-white Flows

	A	B	C	F	G	CAL.	U.S.
A	.9174	.0059	.0044	.0171	.0026	.0162	.0364
B	.0660	.8341	.0052	.0190	.0027	.0439	.0291
C	.0245	.0031	.8792	.0156	.0016	.0376	.0384
F	.0062	.0009	.0011	.9437	.0030	.0143	.0308
G	.0166	.0011	.0012	.0444	.8425	.0182	.0760
CAL.	.0234	.0048	.0095	.0356	.0058	.8728	.0481
U.S.	.0016	.0001	.0002	.0045	.0007	.0011	.9918

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0371	.0033	.0060	.1023	.0073	.0273	.8167)

TABLE 3. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY AGE GROUP

A. 1955-1960 Flows for Age Group #2: 5 to 9 years

	A	B	C	F	G	CAL.	U.S.
A	.8458	.0215	.0074	.0182	.0056	.0384	.0631
B	.0469	.8238	.0054	.0158	.0044	.0473	.0564
C	.0256	.0063	.8101	.0147	.0035	.0690	.0708
F	.0081	.0046	.0032	.8834	.0083	.0346	.0578
G	.0116	.0045	.0018	.0358	.7997	.0245	.1221
CAL.	.0215	.0101	.0113	.0338	.0081	.8497	.0655
U.S.	.0016	.0005	.0004	.0050	.0014	.0025	.9886

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0223	.0095	.0064	.0573	.0109	.0408	.8527)

B. 1955-1960 Flows for Age Group #4: 15 to 19 years

	A	B	C	F	G	CAL.	U.S.
A	.6952	.0424	.0146	.0359	.0111	.0761	.1247
B	.0740	.7221	.0086	.0250	.0069	.0745	.0889
C	.0433	.0107	.6782	.0249	.0059	.1170	.1200
F	.0142	.0081	.0057	.7952	.0146	.0607	.1015
G	.0223	.0086	.0035	.0690	.6136	.0475	.2355
CAL.	.0334	.0167	.0168	.0518	.0123	.7668	.1022
U.S.	.0033	.0011	.0008	.0106	.0030	.0052	.9760

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0228	.0125	.0075	.0666	.0116	.0533	.8255)

TABLE 4. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY AGE GROUP

A. 1955-1960 Flows for Age Group #5: 20 to 24 years

	A	B	C	F	G	CAL.	U.S.
A	.7288	.0377	.0130	.0320	.0099	.0676	.1110
B	.0710	.7332	.0082	.0240	.0067	.0715	.0854
C	.0445	.0110	.6693	.0256	.0061	.1202	.1233
F	.0138	.0079	.0056	.8006	.0142	.0590	.0989
G	.0204	.0079	.0032	.0630	.6468	.0435	.2152
CAL.	.0324	.0161	.0164	.0531	.0271	.7668	.1025
U.S.	.0036	.0012	.0009	.0116	.0033	.0057	.9737

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0272	.0138	.0078	.0735	.0137	.0570	.8070)

B. 1955-1960 Flows for Age Group #8: 35 to 39 years

	A	B	C	F	G	CAL.	U.S.
A	.8825	.0163	.0056	.0138	.0043	.0294	.0481
B	.0391	.8531	.0045	.0132	.0037	.0394	.0470
C	.0215	.0053	.8403	.0124	.0029	.0581	.0595
F	.0063	.0036	.0025	.9097	.0064	.0267	.0448
G	.0087	.0033	.0013	.0268	.8497	.0186	.0916
CAL.	.0186	.0086	.0099	.0291	.0069	.8701	.0568
U.S.	.0016	.0005	.0004	.0050	.0014	.0025	.9886

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0283	.0109	.0074	.0713	.0140	.0455	.8226)

TABLE 5. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY AGE GROUP

A. 1955-1960 Flows for Age Group #11: 50 to 54 years

	A	B	C	F	G	CAL.	U.S.
A	.9293	.0098	.0034	.0083	.0026	.0177	.0289
B	.0256	.9038	.0030	.0086	.0024	.0258	.0308
C	.0130	.0032	.9034	.0075	.0018	.0351	.0360
F	.0043	.0025	.0017	.9378	.0044	.0185	.0308
G	.0053	.0020	.0008	.0163	.9090	.0111	.0555
CAL.	.0137	.0062	.0074	.0205	.0050	.9062	.0410
U.S.	.0008	.0003	.0002	.0026	.0007	.0013	.9941

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0252	.0094	.0067	.0557	.0123	.0341	.8566)

B. 1955-1960 Flows for Age Group #14: 65 to 69 years

	A	B	C	F	G	CAL.	U.S.
A	.9383	.0086	.0030	.0073	.0023	.0152	.0253
B	.0228	.9145	.0027	.0077	.0021	.0228	.0274
C	.0108	.0026	.9195	.0063	.0015	.0292	.0301
F	.0038	.0022	.0015	.9456	.0039	.0160	.0270
G	.0046	.0018	.0007	.0142	.9206	.0097	.0484
CAL.	.0116	.0052	.0060	.0175	.0042	.9206	.0349
U.S.	.0007	.0002	.0002	.0021	.0006	.0011	.9951

Equilibrium Vector:

	A	B	C	F	G	CAL.	U.S.
a =	(.0245	.0083	.0069	.0525	.0119	.0336	.8622)

TABLE 6. TRANSITION MATRICES AND EQUILIBRIUM DISTRIBUTIONS FOR CALIFORNIA: BY SMSA AND NON-SMSA FLOWS

A. Total Flows 1955-1960

	S.	N.S.	U.S.
SMSA	.9167	.0211	.0622
NON-SMSA	.1120	.8218	.0662
U.S.	.0114	.0013	.9873

Equilibrium Vector:

	S.	N.S.	U.S.
a =	(.1451	.0232	.8317)

B. White Flows 1955-1960

	S.	N.S.	U.S.
SMSA	.9135	.0221	.0644
NON-SMSA	.1121	.8214	.0665
U.S.	.0119	.0014	.9867

	S.	N.S.	U.S.
a =	(.1459	.0246	.8295)

C. Non-white Flows 1955-1960

	S.	N.S.	U.S.
SMSA	.9556	.0085	.0359
NON-SMSA	.1101	.8326	.0573
U.S.	.0077	.0005	.9918

	S.	N.S.	U.S.
a =	(.1695	.0111	.8194)

D. Flows for 15-19 Age Group 1955-1960

	S.	N.S.	U.S.
SMSA	.8422	.0391	.1187
NON-SMSA	.1788	.7150	.1062
U.S.	.0215	.0025	.9760

Equilibrium Vector:

	S.	N.S.	U.S.
a =	(.1436	.0270	.8294)

E. Flows for 35-39 Age Group 1955-1960

	S.	N.S.	U.S.
SMSA	.9319	.0175	.0506
NON-SMSA	.0995	.8416	.0589
U.S.	.0102	.0012	.9886

	S.	N.S.	U.S.
a =	(.1571	.0236	.8193)

F. Flows for 50-54 Age Group 1955-1960

	S.	N.S.	U.S.
SMSA	.9554	.0114	.0332
NON-SMSA	.0708	.8873	.0419
U.S.	.0053	.0006	.9941

	S.	N.S.	U.S.
a =	(.1293	.0176	.8530)

TABLE 7. MEAN FIRST PASSAGE TIMES: BY TIME PERIOD

A. 1935-1940 Total Flows

	A	B	C	F	G	CAL.	U.S.
A	25.8	1304.2	1174.6	145.0	915.1	146.5	33.3
B	179.0	200.0	1161.7	145.5	915.0	127.2	34.1
C	215.8	1339.2	188.7	144.4	913.5	106.1	33.0
F	291.9	1400.9	1239.8	14.5	889.8	171.0	29.5
G	301.7	1416.0	1237.1	136.2	153.8	192.1	23.4
CAL.	238.1	1329.2	1154.7	128.6	898.8	20.9	32.2
U.S.	382.1	1492.7	1333.5	216.4	990.3	264.1	1.2

B. 1955-1960 Total Flows

	A	B	C	F	G	CAL.	U.S.
A	39.5	485.2	749.8	119.3	398.0	118.1	16.3
B	204.9	93.5	753.8	119.4	399.3	110.6	16.6
C	238.0	530.2	142.9	120.8	402.2	102.1	15.4
F	264.1	541.2	771.3	15.0	389.0	121.3	16.8
G	271.5	554.3	791.3	116.1	86.2	143.1	11.2
CAL.	243.4	521.9	737.6	109.8	392.4	21.9	15.8
U.S.	300.1	583.8	817.2	143.0	421.7	174.5	1.2

TABLE 8. MEAN FIRST PASSAGE TIMES: BY COLOR

A. 1955-1960 White Flows

	A	B	C	F	G	CAL.	U.S.
A	40.2	460.5	720.6	116.4	376.6	111.2	15.7
B	199.4	89.3	725.0	116.4	378.0	105.1	16.2
C	230.8	505.9	140.8	117.8	380.8	96.6	15.0
F	255.2	515.8	740.7	15.1	367.9	114.5	16.3
G	262.7	529.1	760.8	113.2	82.6	135.6	10.9
CAL.	235.6	497.5	709.0	107.0	371.3	21.0	15.4
U.S.	289.1	557.6	785.4	138.4	399.0	165.1	1.2

B. 1955-1960 Non-white Flows

	A	B	C	F	G	CAL.	U.S.
A	27.0	1762.5	1335.1	135.9	846.2	248.0	27.1
B	190.1	303.0	1319.7	130.2	841.2	204.0	27.9
C	257.2	1810.4	166.7	136.0	851.1	207.8	26.3
F	309.6	1866.5	1388.2	9.8	838.8	252.4	29.1
G	303.5	1876.0	1400.6	123.7	137.0	270.4	20.7
CAL.	272.0	1808.0	1305.8	121.8	830.2	36.6	25.0
U.S.	363.0	1937.6	1455.0	180.3	890.0	339.3	1.2

TABLE 9. MEAN FIRST PASSAGE TIMES: BY AGE GROUP

A. 1955-1960 Flows for Age Group #5: 20 to 24 years

	A	B	C	F	G	CAL.	U.S.
A	36.8	243.3	373.2	59.3	198.3	58.7	9.2
B	103.1	72.5	375.9	59.9	199.5	55.6	10.0
C	119.1	265.8	128.2	60.1	200.5	50.8	8.9
F	132.0	271.3	383.7	13.6	193.9	60.4	9.6
G	135.3	277.7	393.3	57.6	73.0	71.0	6.5
CAL.	122.5	261.9	368.8	54.8	195.8	17.5	9.4
U.S.	148.5	291.7	404.8	70.0	209.1	85.9	1.2

B. 1955-1960 Flows for Age Group #14: 65 to 69 years

	A	B	C	F	G	CAL.	U.S.
A	40.8	1267.0	1733.8	307.3	1011.1	298.7	35.3
B	497.2	120.5	1740.3	305.3	1013.4	277.8	34.2
C	580.2	1385.6	145.0	310.5	1022.2	257.7	32.8
F	641.0	1408.7	1781.9	19.0	986.1	303.6	34.2
G	664.6	1448.9	1833.5	301.8	84.0	363.0	25.7
CAL.	589.2	1361.9	1702.0	280.9	995.2	29.8	31.4
U.S.	736.6	1529.5	1893.7	373.0	1078.2	443.2	1.2

TABLE 10. INTERREGIONAL DISTANCES*

	A	B	C	F	G	CAL.	U.S.
A	--	48	89	403	522	--	--
B	48	--	125	366	485	--	--
C	89	125	--	383	502	--	--
F	403	366	383	--	120	--	--
G	522	485	502	120	--	--	--
CAL.	--	--	--	--	--	--	--
U.S.	--	--	--	--	--	--	--

* County seat to county seat highway mileages.

TABLE 11. CORRELATIONS BETWEEN INTERREGIONAL MEAN FIRST PASSAGE TIMES AND INTERREGIONAL DISTANCES*

	R
Temporal:	
1935-1940 matrix	.024
1955-1960 matrix	-.012
Color:	
White	-.015
Non-white	-.047
Age:	
20- to 24-year age group	-.014
65- to 69-year age group	-.005

* Computed on the basis of twenty observations.